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## G Asymptotic Behavior of Solutions

In the application of linear systems theory to mechanical problems, we have encountered the equation

$$(2) \quad y'' + \gamma y' + \alpha y = f(t),$$

where  $\gamma$  and  $\alpha$  are positive constants with  $\gamma^2 < 4\alpha$  and  $f(t)$  is a forcing function for the system. In many cases it is important for the design engineer to know that a bounded forcing function gives rise only to bounded solutions. More specifically, how does the behavior of  $f(t)$  for large values of  $t$  affect the asymptotic behavior of the solution? To answer this question, do the following:

(a) Show that the homogeneous equation associated with equation (2) has two linearly independent solutions given by

$$e^{a_1 t} \cos \beta t, \quad e^{a_1 t} \sin \beta t,$$

$$\text{where } a_1 = -\gamma/2 < 0 \text{ and } \beta = \frac{1}{2}\sqrt{4\alpha - \gamma^2}.$$

(b) Let  $f(t)$  be a continuous function defined on the interval  $[0, \infty)$ . Use the variation of parameters formula to show that any solution to (2) on  $[0, \infty)$  can be expressed in the form

$$(2) \quad y(t) = c_1 e^{a_1 t} \cos \beta t + c_2 e^{a_1 t} \sin \beta t \\ - \frac{1}{\beta} e^{a_1 t} \cos \beta t \int_0^t f(\tau) e^{-a_1 \tau} \sin \beta \tau d\tau \\ + \frac{1}{\beta} e^{a_1 t} \sin \beta t \int_0^t f(\tau) e^{-a_1 \tau} \cos \beta \tau d\tau.$$

(c) Assuming that  $f$  is bounded on  $[0, \infty)$  (that is, there exists a constant  $K$  such that  $|f(t)| \leq K$  for all  $t \geq 0$ ), use the triangle inequality and other properties of the absolute value to show that  $y(t)$  given in (2) satisfies

$$|y(t)| \leq (|c_1| + |c_2|) e^{a_1 t} + \frac{2K}{|\beta|} (1 - e^{a_1 t})$$

for all  $t > 0$ .

(d) In a similar fashion, show that if  $f_1(t)$  and  $f_2(t)$  are two bounded continuous functions on  $[0, \infty)$  such that  $|f_1(t) - f_2(t)| \leq \epsilon$  for all  $t > t_0$ , and if  $\phi_1$  is a solution to (2) with  $f = f_1$  and  $\phi_2$  is a solution to (2) with  $f = f_2$ , then

$$|\phi_1(t) - \phi_2(t)| \leq M e^{a_1 t} + \frac{2\epsilon}{|\beta|} (1 - e^{a_1 t})$$

for all  $t > t_0$ , where  $M$  is a constant that depends on  $\phi_1$  and  $\phi_2$  but not on  $t$ .

(e) Now assume  $f(t) \rightarrow F_0$  as  $t \rightarrow \infty$ , where  $F_0$  is a constant. Use the result of part (d) to prove that any solution  $\phi$  to (2) must satisfy  $\phi(t) \rightarrow F_0/\alpha$  as  $t \rightarrow \infty$ . [Hint: Choose  $f_1 = f$  and  $f_2 = F_0/\alpha$ .]

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