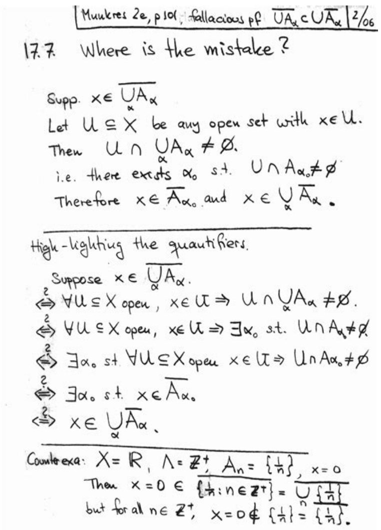

Munkres 2e, p 101, fallacious pf $\overline{\bigcup A_\alpha} \subset \bigcup \overline{A_\alpha}$ | 2/06

17.7 Where is the mistake?

Supp. $x \in \overline{\bigcup_\alpha A_\alpha}$

Let $U \subseteq X$ be any open set with $x \in U$.

Then $U \cap \bigcup_\alpha A_\alpha \neq \emptyset$.

i.e. there exists $\alpha_0$ s.t. $U \cap A_{\alpha_0} \neq \emptyset$

Therefore $x \in \overline{A_{\alpha_0}}$ and $x \in \bigcup_\alpha \overline{A_\alpha}$.

High-lighting the quantifiers.

Suppose $x \in \overline{\bigcup_\alpha A_\alpha}$.

$\overset{?}{\Leftrightarrow} \forall U \subseteq X$ open, $x \in U \Rightarrow U \cap \bigcup_\alpha A_\alpha \neq \emptyset$.

$\overset{?}{\Leftrightarrow} \forall U \subseteq X$ open, $x \in U \Rightarrow \exists \alpha_0$ s.t. $U \cap A_{\alpha_0} \neq \emptyset$.

$\overset{?}{\Leftrightarrow} \exists \alpha_0$ s.t. $\forall U \subseteq X$ open $x \in U \Rightarrow U \cap A_{\alpha_0} \neq \emptyset$

$\overset{?}{\Leftrightarrow} \exists \alpha_0$ s.t. $x \in \overline{A_{\alpha_0}}$

$\overset{?}{\Leftrightarrow} x \in \bigcup_\alpha \overline{A_\alpha}$.

Counterexa: $X = \mathbb{R}$, $\Lambda = \mathbb{Z}^+$, $A_n = \{\frac{1}{n}\}$, $x = 0$

Then $x = 0 \in \overline{\{\frac{1}{n} : n \in \mathbb{Z}^+\}} = \overline{\bigcup_n \{\frac{1}{n}\}}$

but for all $n \in \mathbb{Z}^+$, $x = 0 \notin \overline{\{\frac{1}{n}\}} = \{\frac{1}{n}\}$.

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**Solutions Problems Munkres Topology**